

Basic Sine, Cosine, and Tangent Equations

Basic steps for solving $\cos x = a$:

1. Find all the angles on the unit circle (on $[0, 2\pi]$) that satisfy the equation. One of these solutions will be $s = \cos^{-1} a$ and the other will be $2\pi - s = 2\pi - \cos^{-1} a$.
2. Add or subtract multiples of 2π from each angle.

Basic steps for solving $\sin x = a$:

1. Find all the angles on the unit circle (on $[0, 2\pi]$) that satisfy the equation. You can do this by looking at the unit circle (usually this is less confusing) or by working algebraically. If $s = \sin^{-1} a > 0$, one of these solutions will be $s = \sin^{-1} a$ and the other will be $\pi - s = \pi - \sin^{-1} a$. If $s = \sin^{-1} a < 0$, the two solutions are $s + 2\pi = \sin^{-1} a + 2\pi$ and $\pi - s = \pi - \sin^{-1} a$.
2. Add or subtract multiples of 2π from each angle.

Don't let the algebra freak you out! All you are doing is finding all the angles on the unit circle that satisfy the equation and adding $2k\pi$ to each one to form your solution set.

Basic steps for solving $\tan x = a$:

1. Find one angle on the unit circle that satisfies the equation. This will be either $s = \tan^{-1} a$ if this value is positive, or $s + \pi = \tan^{-1} a + \pi$ if $s = \tan^{-1} a$ is negative.
2. Add or subtract multiples of π from each angle. (Remember that the tangent repeats every π instead of every 2π like sine and cosine).

Examples: Find all real numbers that satisfy each equation.

a) $\sin x = 1$

b) $\cos x = 0$

c) $\cos x = -1/2$

d) $\sin x = \sqrt{2}/2$

e) $\tan x = -\sqrt{3}$

f) $\tan x = 1$

g) $\sin x = -.4375$

h) $\cos x = .8913$

Examples: Find all angles in $[0^\circ, 360^\circ]$ that satisfy each equation

a) $\cos x = \sqrt{3}/2$

b) $\tan x = -3.5$

Sometimes, you have to do a bit of algebra before you can use the techniques above.

a) Solve $2\sin \alpha - 1 = 0$ for $0 \leq \alpha \leq 2\pi$.

b) Solve $3\sin(\beta) + 6 = 5\sin(\beta) + 7$ for $0^\circ \leq \beta \leq 360^\circ$.

We can also solve trigonometric functions involving multiple variables for a specific variable by using the definitions of the inverse functions.

Example: Solve $b = 7 \tan\left(\frac{a}{3}\right) - d$ for a where $-\frac{3\pi}{2} < a < \frac{3\pi}{2}$.